Name solutions

EE 311 Final Exam Fall 2012

December 11, 2012

Closed Text and Notes, No calculators

1) Be sure you have 14 pages and the additional pages of equations.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) Write neatly, if your writing is illegible then print.

5) This exam is worth 150 points.

(5 pts) 1. A 5 C charge is place at the location (1 m, 3 m, -2 m) and it experiences a force of $\mathbf{F}=(20\hat{\mathbf{a}}_x - 5\hat{\mathbf{a}}_y + 10\hat{\mathbf{a}}_z)\mathrm{N}$. What is the electric field intensity at (1 m, 3 m, -2 m)?

$$\vec{E} = \frac{\vec{F}}{Q} = \frac{(20\hat{a}_x - 5\hat{a}_y + 10\hat{a}_z)N}{5C}$$

$$\vec{E} = (4\hat{a}_x - 1\hat{a}_y + 2\hat{a}_z)\frac{N}{C} = (4\hat{a}_x - 1\hat{a}_y + 2\hat{a}_z)\frac{V}{m}$$

$$\left[\frac{N}{C} = \frac{Nm}{Cm} = \frac{J}{Cm} = \frac{V}{m}\right]$$

(18 pts) 2. Circle true or false concerning the statements for a ferromagnetic material.

The Polarization, **P**, is the electric dipole moment per unit volume.	[✓ True]	False
The electric flux density on a spherical surface r = b is the same for a point charge Q located at the origin and for charge Q uniformly distributed on surface r =a where a < b.	[✓ True]	False
If the current flowing through a wire coil is doubled, everywhere the magnetic field intensity is halved.	True	[✓ False]
In cylindrical coordinates, a unit normal vector to the plane $\phi = 45^\circ$ is $\mathbf{a}_\rho$	True	[✓ False]
The intersection of the surfaces r = 1m and $\theta = \frac{\pi}{3}$ is a circle.	[✓ True]	False
The force between two parallel wires with current flowing in the same direction is repulsive.	True	[✓ False]
The direction of the electric field is always parallel to an equipotential surface.	True	[✓ False]
The force on a moving charge in a magnetic field does no work.	[✓ True]	False
Dielectric breakdown is the onset of conduction, which occurs at high lectric field intensities.	[✓ True]	False

(12 pts) 3. A conducting spherical shell of radius 10^9 m contains a charge of 1 C. If $V(\infty)=0$, what is $V(100\,m, \pi, \frac{\pi}{2})$?

$$\vec{E} = \frac{1\,C}{4\pi\epsilon_0 r^2}\hat{a}_r \qquad \text{for} \quad r > 10^9\,m$$

$$= 0 \qquad \text{for} \quad r < 10^9\,m$$

$$V(100\,m) - V(\infty) = -\int_{\infty}^{10^9} \frac{1\,C}{4\pi\epsilon_0 r^2}\,dr - \int_{10^9}^{100} 0 \cdot dr$$

$$V(100\,m) - 0 = -\int_{\infty}^{10^9} \frac{1\,C}{4\pi\epsilon_0 r^2}\,dr = \frac{1\,C}{4\pi\epsilon_0 r}\Big|_{\infty}^{10^9}$$

$$V(100m) = \frac{1\,C}{4\pi\epsilon_0(10^9 m)} = \frac{1\,C}{4\pi\left(\frac{10^{-9}}{36\pi}\frac{F}{m}\right)(10^9 m)}$$

$$= 9\frac{C}{F} = 9\frac{C}{\left(\frac{C}{V}\right)}$$

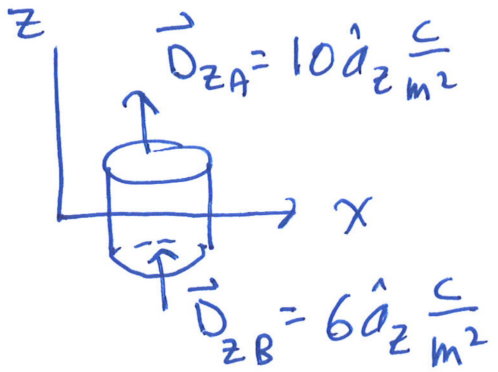
$$= 9V$$

(5 pts). 4. Point charges 30 nC, -20 nC, and 10 nC are located at (-1, 0, 2), (0, 0, 0) and (1, 5, -1). What is the value of $\oint \mathbf{D} \bullet d\mathbf{S}$ over the surface of a cube of side 6 m centered at the origin?

10 nC

(10 pts) 5. For $z > 0$ $\mathbf{D}=6\hat{\mathbf{a}}_x+3\hat{\mathbf{a}}_y+10\hat{\mathbf{a}}_z \frac{C}{m^2}$ and the permittivity is $\varepsilon=3\varepsilon_o$. For $z < 0$ $\mathbf{D}=D_x\hat{\mathbf{a}}_x+D_y\hat{\mathbf{a}}_y+6\hat{\mathbf{a}}_z \frac{C}{m^2}$ and $\varepsilon=2\varepsilon_o$.

(4 pts) a) What is the charge density on the z = 0 plane?



$$D_{zA} - D_{zB} = \rho_s$$

$$10\frac{C}{m^2} - 6\frac{C}{m^2} = \rho_s$$

$$\rho_s = 4\frac{C}{m^2}$$

(6 pts) b) Find D_x and D_y .

$$E_{Ax} = E_{Bx}$$

$$\frac{D_{Ax}}{3\epsilon_o} = \frac{D_{Bx}}{2\epsilon_o}$$

$$\frac{6\frac{C}{m^2}}{3\epsilon_o} = \frac{D_{Bx}}{2\epsilon_o}$$

$$D_{Bx} = 4\frac{C}{m^2}$$

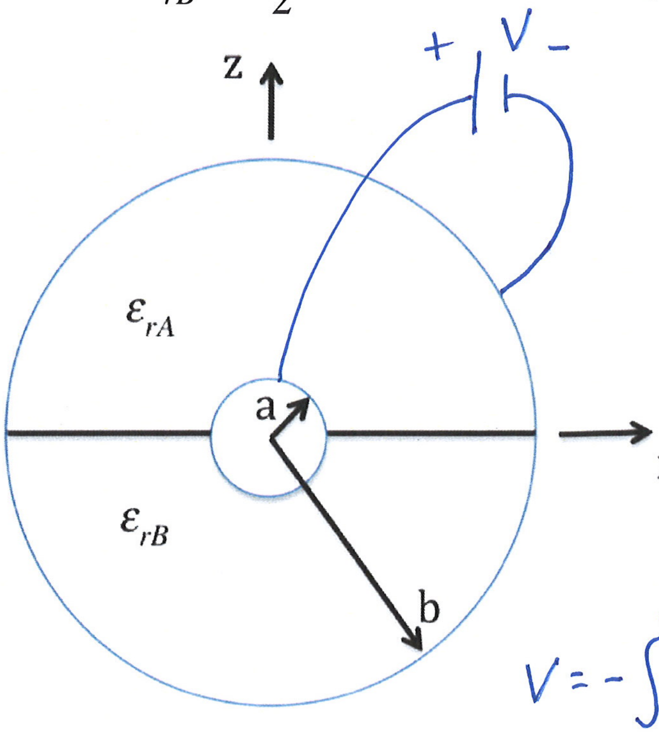
$$E_{Ay} = E_{By}$$

$$\frac{D_{Ay}}{3\epsilon_o} = \frac{D_{By}}{2\epsilon_o}$$

$$\frac{3\frac{C}{m^2}}{3\epsilon_o} = \frac{D_{By}}{2\epsilon_o}$$

$$D_{By} = 2\frac{C}{m^2}$$

(15 pts) 6. A capacitor is formed from two concentric spheres. The radius of the inner sphere is a, the radius of the outer sphere is b, and with dielectrics ε_{rA} for $0<\theta<\frac{\pi}{2}$ and ε_{rB} for $\frac{\pi}{2}<\theta<\pi$. What is the capacitance?



$$V = -\int_b^a \vec{E} \cdot \vec{dl}$$

so $\vec{E}$ must be independent of θ and ϕ

$$\vec{E} = \frac{A}{4\pi\epsilon_0 r^2}\hat{a}_r$$

$$V = -\int_b^a \frac{A}{4\pi\epsilon_0 r^2}\hat{a}_r \cdot dr\,\hat{a}_r = -\frac{A}{4\pi\epsilon_0}\int_b^a \frac{dr}{r^2}$$

$$V = \frac{A}{4\pi\epsilon_0}\frac{1}{r}\Big|_b^a = \frac{A}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$\vec{D} = \epsilon_{rA}\epsilon_0\frac{A}{4\pi\epsilon_0 r^2}\hat{a}_r = \frac{A\epsilon_{rA}}{4\pi r^2}\hat{a}_r \qquad 0<\theta<\frac{\pi}{2}$$

$$\vec{D} = \epsilon_{rB}\epsilon_0\frac{A}{4\pi\epsilon_0 r^2}\hat{a}_r = \frac{A\epsilon_{rB}}{4\pi r^2}\hat{a}_r \qquad \frac{\pi}{2}<\theta<\pi$$

From the electric flux densities, the surface charge densities on the inner sphere are

$$\rho_s = \frac{A\epsilon_{rA}}{4\pi a^2} \qquad 0<\theta<\frac{\pi}{2}$$

$$= \frac{A\epsilon_{rB}}{4\pi a^2} \qquad \frac{\pi}{2}<\theta<\pi$$

The surface area of the inner sphere is

$$4\pi r^2 = 4\pi a^2$$

The total charge on the inner sphere is,

$$Q = \frac{A\,\epsilon_{rA}}{4\pi a^2}\underbrace{2\pi a^2}_{\substack{\text{half the}\\ \text{surface area}\\ \text{of inner}\\ \text{sphere}}} + \frac{A\,\epsilon_{rB}}{4\pi a^2}\,2\pi a^2$$

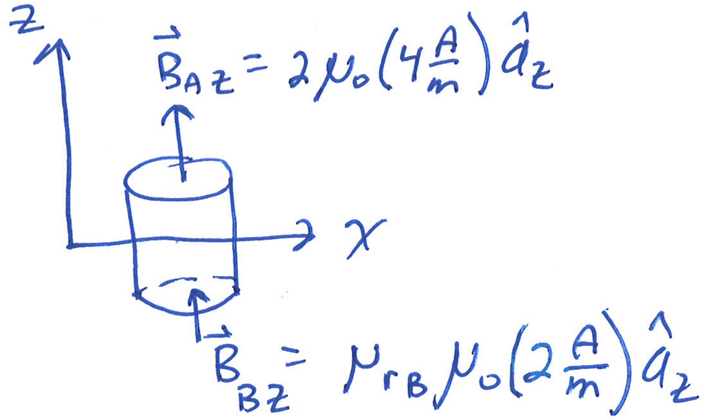
$$Q = \frac{A}{2}\left(\epsilon_{rA} + \epsilon_{rB}\right)$$

$$C = \frac{Q}{V} = \frac{\frac{A}{2}\left(\epsilon_{rA}+\epsilon_{rB}\right)}{\frac{A}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)} = 2\pi\epsilon_0\,\frac{\epsilon_{rA}+\epsilon_{rB}}{\left(\frac{b-a}{ab}\right)}$$

$$= \frac{2\pi\epsilon_0\,ab\left(\epsilon_{rA}+\epsilon_{rB}\right)}{b-a}$$

(10 pts) 7. For $z > 0$ $\mathbf{H} = 4\hat{\mathbf{a}}_x + 4\hat{\mathbf{a}}_y + 4\hat{\mathbf{a}}_z \frac{\mathrm{A}}{\mathrm{m}}$ and the permeability is $\mu = 2\mu_o$. For $z < 0$ $\mathbf{H} = 1\hat{\mathbf{a}}_x + 4\hat{\mathbf{a}}_y + 2\hat{\mathbf{a}}_z \frac{\mathrm{A}}{\mathrm{m}}$

(5 pts) a) What is the permeability for z < 0?



$\vec{B}_{Az} = 2\mu_o(4\frac{A}{m})\hat{a}_z$

$\vec{B}_{Bz} = \mu_{rB}\mu_o(2\frac{A}{m})\hat{a}_z$

$B_{Az} = B_{Bz}$

$2\mu_o(4\frac{A}{m}) = \mu_{rB}\mu_o(2\frac{A}{m})$

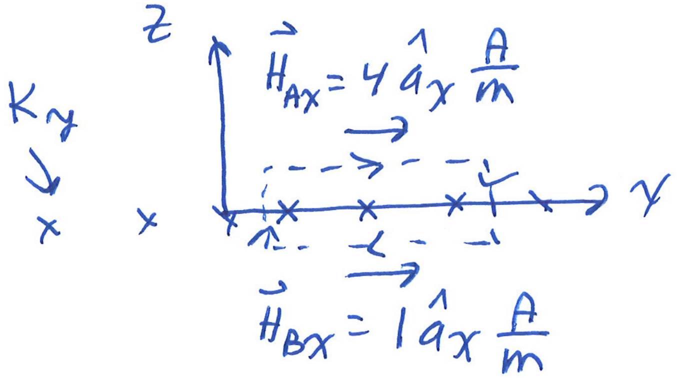
$\mu_{rB} = 4$

or

$\mu_B = 4\mu_o$

(5 pts) b) What current is flowing on the z = 0 plane?

$H_{Ay} = H_{By}$ so $K_x = 0$



$\vec{H}_{Ax} = 4\hat{a}_x \frac{A}{m}$

$\vec{H}_{Bx} = 1\hat{a}_x \frac{A}{m}$

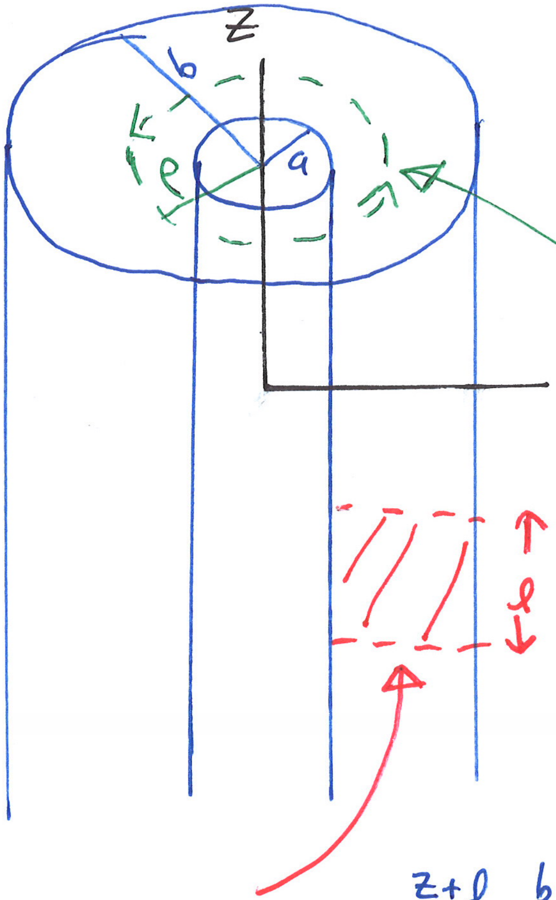
$\oint \vec{H}\cdot d\vec{l} = I_{encl} = K_y w$

$4w - 1w = K_y w$

$K_y = 3$

$\vec{K} = 3\frac{A}{m}\hat{a}_y$

(10 pts) 8. From fundamental field concepts, determine the inductance per unit length for a coaxial cable whose inner conductor has radius a, outer conductor radius b, and dielectric of permittivity ε and permeability μ between the conductors.



Assume a current I flowing in the $\hat{a}_z$ direction on the inner conductor

$$\oint \vec{H}\cdot\vec{d\ell} = I \quad \text{for } a<\rho<b$$

$$H\,2\pi\rho = I$$

$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$$

$$\vec{B} = \frac{\mu I}{2\pi\rho}\hat{a}_\phi \,,\ a<\rho<b$$

ψ through this surface $= \int_z^{z+\ell}\int_a^b \vec{B}\cdot\vec{ds} = \int_z^{z+\ell}\int_a^b \frac{\mu I}{2\pi\rho}\hat{a}_\phi\cdot d\rho\, dz\,\hat{a}_\phi$

$$= \frac{\mu I}{2\pi}\int_a^b \frac{d\rho}{\rho}\int_z^{z+\ell} dz = \frac{\mu I\ell}{2\pi}\int_a^b \frac{d\rho}{\rho}$$

$$= \frac{\mu I\ell}{2\pi}\ln\rho\,\Big|_a^b = \frac{\mu I\ell}{2\pi}(\ln b-\ln a) = \frac{\mu I\ell}{2\pi}\ln\frac{b}{a}$$

$$L = \frac{\psi}{I} = \frac{\mu\ell}{2\pi}\ln\frac{b}{a}$$

inductance per unit length $= \frac{L}{\ell} = \frac{\mu}{2\pi}\ln\frac{b}{a}$

(8 pts) 9. Fill in the table with the standard units for the following

Magnetic flux density, **B**	$\frac{Wb}{m^2}$ or T
Magnetic field intensity, **H**	$\frac{A}{m}$
Electric Field Intensity, **E**	$\frac{V}{m}$
Electric Flux Density, **D**	$\frac{C}{m^2}$
Polarization, **P**	$\frac{C}{m^2}$
Magnetization, **M**	$\frac{A}{m}$
Electric flux, Ψ	C
Magnetic flux, Ψ	Wb

(5 pts) 10. The integral, $\oint \mathbf{H} \cdot \mathbf{dS}$, over a closed surface will not be zero when

A) the surface encloses only free space.

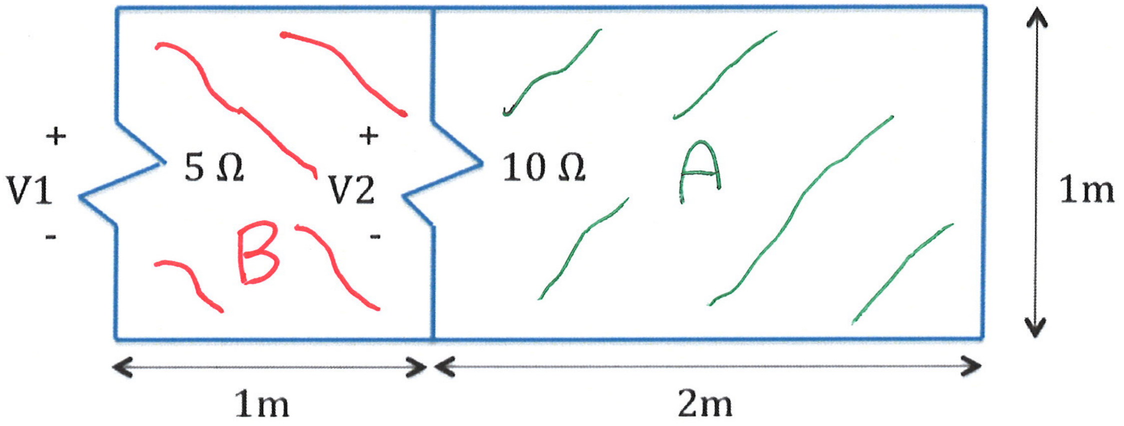
[✓ B)] the surface encloses the end of a magnet.

C) the surface is entirely inside a ferromagnetic material.

D) actually it will always be equal to zero.

(10 pts) 11. The two-resistor circuit is in the field of magnitude $B=10t\frac{Wb}{m^2}$ that is into the page.

Determine V1 and V2.



$$\oint \vec{E}\cdot\vec{dl} = -V_2 = -\frac{d\psi}{dt}_{A} = -\frac{d}{dt}\left[\left(10t\frac{Wb}{m^2}\right)2m^2\right]$$

cw around A

$$-V_2 = -20V$$

$$V_2 = 20V$$

$$\oint \vec{E}\cdot\vec{dl} = -V_1 = -\frac{d\psi}{dt}_{A+B} = -\frac{d}{dt}\left[\left(10t\frac{Wb}{m^2}\right)3m^2\right]$$

cw around
A + B

$$-V_1 = -30V$$

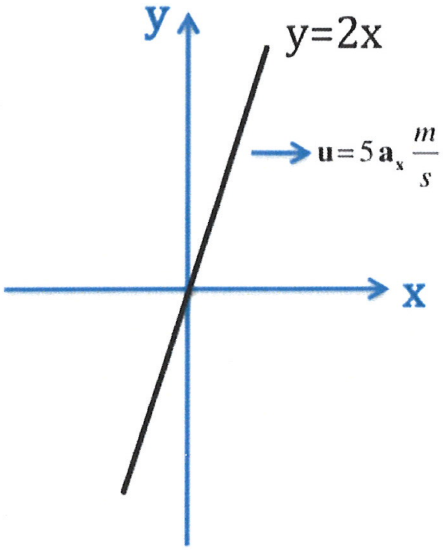
$$V_1 = 30V$$

(4 pts) 12. A parallel plate capacitor has plate area 0.1 m^2 and plate separation of 0.1 mm. If there is a displacement current density of 5 $\frac{\mu A}{m^2}$ between the plates, how much charge is flowing onto the capacitor per second?

$$I = I_D = J_D S = \left(5 \frac{\mu A}{m^2}\right)(0.1 m^2) = 0.5 \mu A$$

0.5×10^{-6} C is flowing onto the plate per second

(5 pts). 13. Shown is the line y = 2x. Write the equation to represent this line moving with velocity $\mathbf{u} = 5\mathbf{a}_x \frac{m}{s}$.



replace x with

$x - (5 \frac{m}{s}) t$

$$y = 2\left[x - (5 \tfrac{m}{s}) t\right]$$

(5 pts) 14. An electromagnetic wave consists of

[✓ A)] Perpendicular oscillating electric and magnetic fields

B) Perpendicular stationary electric and magnetic fields

C) Parallel oscillating electric and magnetic fields

D) Parallel stationary electric and magnetic fields

E) Anti-parallel oscillating electric and magnetic fields

F) Anti-parallel stationary electric and magnetic fields

(5 pts) 15. In a right-hand circularly polarized electromagnetic wave

A) there are two perpendicular propagating electric fields that are in phase.

B) there is a propagating polarization field, P, which is perpendicular to the propagating electric field, E, and they are in phase.

[✓ C)] there are two perpendicular electric fields, which are phase shifted by 90 degrees.

D) the electric and magnetic field waves are phase-shifted by 90 degrees.

(12 pts). 16. What are the standing wave ratios of the following,

(4 pts) A) the standing wave $E(z,t) = 5\sin[(3\times10^{8}s^{-1})t]\sin[(1m^{-1})z]\hat{a}_x \ \frac{V}{m}$

$$S = \frac{E_{max}}{E_{min}} = \frac{5}{0} = \infty$$

(4 pts) B) the propagating wave $E(z,t) = 10\cos[(3\times10^{8}s^{-1})t - (1m^{-1})z]\hat{a}_x \ \frac{V}{m}$

$$S = \frac{E_{max}}{E_{min}} = \frac{10}{10} = 1$$

(4 pts) C) $E(z,t) = \{10\cos[(3\times10^{8}s^{-1})t - (1m^{-1})z] + 5\sin[(3\times10^{8}s^{-1})t]\sin[(1m^{-1})z]\}\,\hat{a}_x \ \frac{V}{m}$

$$S = \frac{E_{max}}{E_{min}} = \frac{15}{10} = 1.5$$

(8 pts) 17 A uniform plane electromagnetic wave has its electric field in the $+\hat{\mathbf{a}}_y$ when its magnetic field is in the $-\hat{\mathbf{a}}_x$-direction. The frequency of the wave is $\omega = 2x10^8\, s^{-1}$. If all of space is filled with a material with $\mu_r = 1$ and $\varepsilon_r = 9$, and if the maximum value of the electric field is $10\ \frac{V}{m}$, write an equation that describes the electric field. The speed of an electromagnetic wave in a vacuum is $3x10^8 \frac{m}{s}$.

$$\vec{\mathcal{P}} = \vec{E} \times \vec{H} \qquad \hat{a}_y \times (-\hat{a}_x) = +\hat{a}_z$$

so the wave is propagating in the $-\hat{a}_z$ direction

$$u = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_r \epsilon_r}\sqrt{\mu_o \epsilon_o}} = \frac{1}{\sqrt{9}\sqrt{\mu_o \epsilon_o}}$$

$$= \frac{3 \times 10^8 \frac{m}{s}}{\sqrt{9}} = 1 \times 10^8 \frac{m}{s}$$

$$u = \frac{\omega}{\beta} = \frac{2 \times 10^8 s^{-1}}{\beta} = 1 \times 10^8 \frac{m}{s}$$

$$\beta = 2\ m^{-1}$$

$$\vec{E}(z,t) = 10 \frac{V}{m} \cos\left[(2 \times 10^8 s^{-1})t - (2 m^{-1}) z\right] \hat{a}_y$$

(8 pts) 18. The region z < 0 is free space and the region z > 0 is a lossless dielectric with $\mu_r = 1$ and $\varepsilon_r = 4$. The electric field of a TEM wave propagating in the free space region is described by $\mathrm{E}(z,t) = 9\cos[(3\times10^8 \mathrm{s}^{-1})t - (1\mathrm{m}^{-1})z]\hat{\mathbf{a}}_x \ \frac{\mathrm{V}}{\mathrm{m}}$. Determine the equation describing the electric field for region z > 0.

Let $z<0$ be region 1 and $z>0$ be region 2

$$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_0}} \qquad \eta_2 = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{1\mu_0}{4\epsilon_0}} = \frac{1}{2}\sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{1}{2}\eta_1$$

$$\tau = \frac{2\eta_2}{\eta_2+\eta_1} = \frac{2\left(\frac{\eta_1}{2}\right)}{\frac{\eta_1}{2}+\eta_1} = \frac{1}{\left(\frac{3}{2}\right)} = \frac{2}{3}$$

$$E_{t_0} = \tau E_{i_0} = \frac{2}{3}\left(9\frac{V}{m}\right) = 6\frac{V}{m}$$

$$u_2 = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_r\epsilon_r}\sqrt{\mu_0\epsilon_0}} = \frac{3\times10^8\frac{m}{s}}{\sqrt{\mu_r\epsilon_r}}$$

$$= \frac{3\times10^8 \, m/s}{\sqrt{(1)(4)}} = 1.5\times10^8\frac{m}{s}$$

$$u_2 = \frac{\omega}{\beta_2} \qquad \beta_2 = \frac{\omega}{u_2} = \frac{3\times10^8 s^{-1}}{1.5\times10^8\frac{m}{s}} = 2m^{-1}$$

$$\vec{E}_t(z,t) = 6\cos[(3\times10^8 s^{-1})t - (2m^{-1})z]\hat{a}_x \frac{V}{m}$$